JKAU: Sci., **vol. 13,** pp. 31-48 (1421 A.H. / 2001 A.D.)

Determination of Intrinsic Color Indices of Globular Clusters and the Coefficients of Differential Absorption

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ABSTRACT. A simple model for the galactic absorbing layer is used to derive linear representation of the observed color index of a globular cluster. Selection and solution criteria enable us to determine very accurately the mean intrinsic color indices of globular clusters as $(B\text{-}V)_o$ = 0.582344 mag; $(U\text{-}B)_o$ = 0.033909 mag; $(V\text{-}I)_o$ = 0.798231 mag; $(V\text{-} R)_o$ 0.413043 mag. Also, from the slope of such linear representation we compute: (1) the effective thickness of the galactic absorbing layer which turn out to be 200 pc, is the value currently adopted, and (2) the values of the coefficients of differential absorption as $k_u - k_b$ = 0.724 mag/kpc, $k_v - k_i$ = 0.81549 mag/kpc and $k_v - k_r$ = 0.33189 mag/kpc.

Introduction

Globular clusters are considered to be very important since they represent a very interesting tool for studying the early dynamical and chemical evolution of the galactic halo[1].

It seems, that our knowledge of the globular cluster system is as complete as materially possible. In fact, the number of known globular cluster has hardly increased during the last decades. However, theoretical and observational studies regarding their dynamics have witnessed tremendous developments, which mainly took place during the last two decades.

In the search for globular clusters no particular region of the sky nor any particular subgroup has been favored. In this sense it can be stated that the data at our disposal are not distorted or biased by selection effect. In other words, not with standing the fact that our knowledge of the system of globular cluster is limited to a sample, this sample can be considered as sufficiently representative for the whole.

In this article, we have investigated color indices and differential absorption coefficients of the globular cluster system. In the next section, we discuss the model for the thickness of the absorbing layer, where we have chosen a simple one for this purpose utilizing a linear representation of the observed color index of a typical globular cluster. In addition, we look into the differential absorption coefficients of the system, and discuss the solution and criteria in order to determine the mean intrinsic color indices for the globular cluster. Meanwhile, after this section, numerical studies providing an acceptable solution for our case, as well as an application is provided. The effective thickness of the galactic absorping layer is indicated, as well as the values of the coefficients of the differential absorption.

Basic Formulation

Linear Representation of the Observed Color Index

In what follows, linear representation for the observed color index of a globular cluster will be developed using a simple model for the galactic absorping layer[2]. This model assumes that, the layer is thin, plane-parallel, homogeneous, and symmetrical about the galactic plane (Fig. 1).

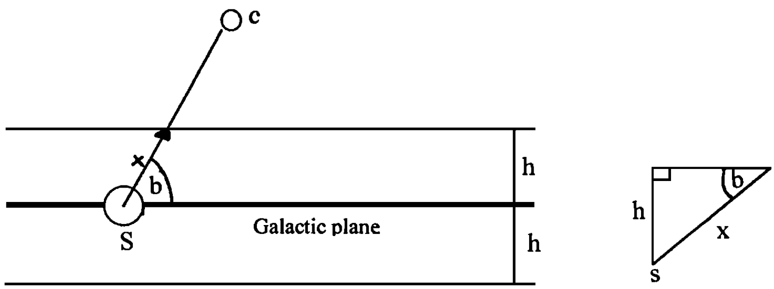

FIG. 1.

Let C be a globular cluster, S the sun supposed at the center of the absorbing layer, h half of its thickness, and b is the galactic latitude of the cluster. Under these assumptions, the quantity of absorping material traversed by the radiation coming from C depends only on the galactic latitude b, or more precisely, on its absolute value $| b |$.

It is well known that, after traversing a slab of homogeneous absorbing material of thickness x, the incident intensity I_o of a (monochromatic) ray is reduced and becomes

$$I = I_o \exp(-\alpha x)$$

where α is the absorption coefficient of the material. If the corresponding magnitude difference is *dm,* then

$$dm = -2.5 \log I/I_o = k\,x \qquad (1)$$

where

$$k = 2.5 \;\alpha / \ln 10 \qquad (2)$$

is usually expressed in magnitudes per kiloparsecs.

Now, the path length of the light from the cluster at galactic latitudes b through the layer is $x = h{*}\csc|b|$, consequently

$$dm = k{*}h{*}\csc|b| \qquad (3)$$

Suppose we have measured the magnitudes of a globular cluster which correspond to two different regions of the spectrum (T, S) (say) [e.g.(B, V), (U,B), magnitudes]. Let T_o and S_o the magnitudes as they would be in absence of absorption.

Denote by k_t and k_s respectively the values of the absorption coefficient for the regions of the spectrum to which these magnitudes pertain. Now, according to Equation (3), the observed magnitudes given by:

$$T_{obs} = T_o + k_t{*}h{*} \csc |b|,$$

$$S_{obs} = S_o + k_t{*}h{*} \csc |b|$$

The difference $(T\text{-}S)_{obs}$ of the observed magnitudes is the observed color index, the difference $(T\text{-}S)_o$ defines the intrinsic color index of the cluster. From the two equations we get.

$$(T\text{-}S)_{obs} = (T\text{-}S)_o + \Delta_{ts}{*}h{*} \csc |b|, \qquad (4)$$

where $\Delta_{ts} = k_t - k_s$ is the difference of absorption coefficients for the two spectral regions concerned, or, as it is usually called, the *coefficient of differential absorption* for these two regions of the spectrum. The difference

$$(T\text{-}S)_{obs} - (T\text{-}S)_o = E_{T-S}, \qquad (5)$$

is the color excess.

Now, let us adopt the assumption that, statistically all globular clusters have the same intrinsic color index[3]. Then Equation (4) is a linear representation of the observed color indices $(T\text{-}S)_{obs}$ of the globular clusters with csc b, and can be written as

$$\eta \;C_1 + C_2 \;\xi, \qquad (6)$$

where

$$\eta = (T\text{-}S)_{obs}\,,\ C_1 = (T\text{-}S)_o\,,\ C_2 = \Delta_{ts*}h\,,\ \xi = \csc\ |\,b\,| \tag{7}$$

Determination of C_1 and C_2 and the Error Estimates

Solution for C_1 and C_2

Let N (say) observational data (ξ_k, η_k); then by defining

$$A_1 = \sum_{j=1}^{N} \xi_j\ ;\ A_2 = \sum_{j=1}^{N} \xi_j^2\ ;\ B_1 = \sum_{j=1}^{N} \eta_j\ ;\ B_2 = \sum_{j=1}^{N} \eta_j \xi_j\ ;\ B_3 = \sum_{j=1}^{N} \eta_j^2\ , \tag{8}$$

we get for the solutions of C_1 and C_2 of Equation (6) in the sense of the least-squares criterion, the expressions

$$C_1 = (B_1 A_2 - B_2 A_1) / D\ , \tag{9}$$

$$C_2 = (B_2 N - B_1 A_1) / D\ , \tag{10}$$

where

$$D = N A_2 - A_1^2\ . \tag{11}$$

The error estimates are given in the following points.

Standard Error of the Fit

According to the least-squares criterion, the standard error of the fit is given by

$$\sigma = \left[\sum_{j=1}^{N} \{C_1 + C_2 \xi_j - \eta_j\}^2\ / (N-2) \right]^{1/2}\ .$$

By expanding the above equation, we obtain

$$\sigma = \left[\{B_3 - C_1^2 N - C_1^2\ A_2 - 2C_1 C_2 A_1\} / (N-2)\right]^{1/2}\ , \tag{12}$$

where A_1, A_2, B_3, C_1 and C_2 are given by Equations (2.7), (2.8) and (2.9). Note that if the precision is measured by the random error e, then

$$e = 0.6745\ \sigma\ . \tag{13}$$

Standard Errors of C_1 and C_2

The standard errors of the least-squares solutions C_1 and C_2 are given as

$$\sigma_{C_1} = \sigma \sqrt{g_{11}}\ ,\ \sigma_{C_2} = \sigma \sqrt{g_{22}}\ ,$$

where g_{kk} are the diagonal elements of the inverse of the matrix G used for the solutions C_1 and C_2. Therefore, the standard and the random errors of C_1 and C_2 are given as

$$\sigma_{C_1} = \sigma\sqrt{A_2 / D} \;\; ; \;\; e_{C_1} = 0.6745 \;\; \sigma_{C_1} \;\; , \tag{14}$$

$$\sigma_{C_2} = \sigma\sqrt{N / D} \;\; ; \;\; e_{C_2} = 0.6745 \;\; \sigma_{C_2} \;\; , \tag{15}$$

where A_2, D and σ are given from Equations (7), (10) and (11).

The Average Squared Distance Q

The average squared distance Q between the least-squares estimators of C_1 and C_2 and their true values is given as[4]

$$Q = \sigma^2 \sum_{i=1}^{2} 1/\lambda_i \;,$$

where λ_1 and λ_2 and are the eigen values of the matrix G. Evaluating the λ we obtain

$$Q \;=\; (\sigma_{C_1})^2 \;+\; (\sigma_{C_2})^2 \;. \tag{16}$$

Coefficient of Correlation

Another useful estimator is the correlation coefficient which is a measure of the degree to which the two variables (ξ, η) are related to each other. In our case, the coefficient of correlation r is given by

$$r = C_2 \left(\frac{D}{N B_3 - B_1^2} \right)^{1/2} \;. \tag{17}$$

The closer r is to 1 or –1, the stronger the linear representation between the two variables (ξ, η). The closer r is to zero, the weaker the linear representation. The sign of r indicates the direction of the representation between ξ and η.

Numerical Studies

Acceptable Solution Set

Although the least-squares method is one of the most powerful techniques that could be used for the present problem, it is at the same time exceedingly critical. This is due to the fact that least-squares estimates suffer from the lack of control on the sensitivity of the solution from the optimization criterion (the variance σ^2 set to minimum). As a result, there may exist a situation in which there are many significantly different values of the solutions C_1 and C_2 [see Tables 1-4] that reduce the variance to an acceptably small value. At this stage we should point out that: (1) the accuracy of the estimators and the accuracy of the fitted representation [Equation (6)] are two distinct problems; and (2) an accurate estimator will always produce small variance, but small variance does not guarantee an accurate estimator.

According to these two notes, it is necessary to reformalize the concept of an *acceptable solution for our problem.* This last point is illustrated as follows. Let us define an acceptable solution set for the present problem as

$$S.S = \{C_1 , C_2 : e \le \varepsilon_1 , | e_{C_1} / \mathrm{C}_1 | \le \varepsilon_2 , | e_{C_2} / \mathrm{C}_2 | \le \varepsilon_3 , Q \le \varepsilon_4 , r \ge \varepsilon_5 .\} \quad (18)$$

TABLES 1. Dependence of the solutions and the tolerances on the selection criteria *B-V*.

O-C	No.	C_1	C_2	e_{C_1}	e_{C_2}	e	Q	r	e_{C_1}/C_1	e_{C_2}/C_2
20	116	0.668043	0.05881	0.020482	0.002301	0.151024	0.000933722	0850178	0.030353	0.035749
0.5	110	0.641075	0.062121	0.015012	0.001732	0.108222	0.000501911	0.918832	0.023608	0.027655
0.3	103	0.631722	0.061657	0.012633	0.001504	0.089699	0.0003557	0.939843	0.019561	0.024282
0.2	94	0.612391	0.061885	0.009713	0.001251	0.064981	0.000210815	0.961087	0.016188	0.019561
0.17	84	0.60736	0.061867	0.008228	0.001034	0.053075	0.000151144	0.975735	0.01349	0.016863
0.15	78	0.603873	0.062453	0.007157	0.000906	0.04544	0.000114392	0.982883	0.012141	0.014839
0.12	71	0.594037	0.062539	0.006217	0.000778	0.038114	0.000086294	0.988475	0.010792	0.012411
0.1	69	0.590679	0.062606	0.006057	0.00075	0.036703	0.0000818	0.989534	0.010118	0.012006
0.08	61	0.582135	0.063602	0.005513	0.00074	0.031525	0.000068018	0.991344	0.009443	0.011601
0.07	56	0.582344	0.063278	0.005184	0.000703	0.029089	0.00006016	0.992752	0.008769	0.011129

TABLES 2. Dependence of the solutions and the tolerances on the selection criteria *U-B*.

O-C	No.	C_1	C_2	e_{C_1}	e_{C_2}	e	Q	r	e_{C_1}/C_1	e_{C_2}/C_2
20	108	0.102761	0.052201	0.024795	0.002913	0.171132	0.00137	0.761229	0.241269	0.055849
0.5	103	0.035855	0.061434	0.017368	0.002241	0.113661	0.000674	0.878582	0.484358	0.036423
0.3	95	0.015195	0.063193	0.013708	0.001801	0.08848	0.00042	0.926102	0.898097	0.028464
0.25	88	0.010568	0.066135	0.011772	0.00157	0.073889	0.00031	0.950621	1.052895	0.023742
0.2	84	0.018501	0.067479	0.010987	0.001459	0.067886	0.00027	0.960373	1.290993	0.21584
0.18	76.3	0.010264	0.066901	0.009604	0.001289	0.057934	0.000206	0.971113	0.937555	0.019291
0.16	70	0.007177	0.066399	0.008343	0.001108	0.049429	0.000156	0.979823	1.16014	0.016863
0.14	69	0.023009	0.07005	0.008925	0.001403	0.047322	0.000179	0.971711	0.387838	0.019561
0.12	65	0.033909	0.0724	0.008354	0.001357	0.042263	0.000157	0.976545	0.246193	0.018886

TABLES 3. Dependence of the solutions and the tolerances on the selection criteria *V-I*.

O-C	No.	C_1	C_2	e_{C_1}	e_{C_2}	e	Q	r	e_{C_1}/C_1	e_{C_2}/C_2
20	99	0.936731	0.065355	0.031683	0.004261	0.192928	0.002246	0.724244	0.033725	0.065224
0.5	90	0.891983	0.067573	0.019577	0.002848	0.111244	0.00086	0.862627	0.022259	0.042156
0.3	84	0.853439	0.072516	0.015187	0.002269	0.084029	0.000518	0.921955	0.17807	0.031297
0.2	74	0.834902	0.073722	0.012185	0.001918	0.96544	0.000334	0.950401	0.014569	0.025631
0.15	63	0.823868	0.074086	0.009371	0.001501	0.047491	0.000198	0.973566	0.011332	0.021584
0.1	54	0.819265	0.075309	0.008536	0.001566	0.037962	0.000166	0.976173	0.010387	0.02091
0.07	41	0.801728	0.080273	0.007724	0.001533	0.29916	0.000136	0.98471	0.009645	0.018886
0.06	34	0.798231	0.081549	0.007259	0.001399	0.025829	0.00012	0.98989	0.009106	0.016863

TABLES 4. Dependence of the solutions and the tolerances on the selection criteria *V-R*.

O-C	No.	C_1	C_2	e_{C_1}	e_{C_2}	e	Q	r	e_{C_1}/C_1	e_{C_2}/C_2
20	108	0.1028	0.0522	0.0248	0.0029	0.0651	0.0014	0.7612	0.0241	0.056
0.5	82	0.4564	0.0301	0.0119	0.0015	0.0651	0.0003	0.8323	0.0256	0.0499
0.3	81	0.4626	0.0284	0.0111	0.0014	0.0603	0.0003	0.8336	0.0202	0.0496
0.2	77	0.4368	0.0318	0.0098	0.0014	0.0501	0.0002	0.8773	0.0223	0.0425
0.15	73	0.4267	0.0319	0.0086	0.0012	0.0429	0.0002	0.9085	0.0196	0.0371
0.13	68	0.4154	0.0345	0.007	0.001	0.0339	0.001	0.9403	0.0229	0.0304
0.11	66	0.4122	0.0351	0.0066	0.001	0.0314	1E-04	0.9476	0.0162	0.0283
0.09	62	0.4122	0.035	0.0058	0.0009	0.0274	0.00007	0.9615	0.0142	0.0249
0.07	58	0.4144	0.034	0.0051	0.0008	0.0237	6E-05	0.9665	0.0121	0.0239
0.06	53	0.4159	0.0334	0.0047	0.0008	0.0203	5E-05	0.9728	0.115	0.0223
0.04	46	0.413	0.0332	0.004	0.0006	0.016	4E-05	0.984	0.0094	0.0184

The ε is the different color indices. In this respect, the error controlling formulae of last section may be useful to determine the coefficients C_1 and C_2 very accurately.

Numerical Results

- The data samples were collected from[7]. The total number of globular clusters is 143, while the total number of clusters with complete data (*T-S, b*) are 116, 108, 99 and 108 for the color indices (*B-V*), (*U-B*), (*V-I*), and (*V-R*) respectively. The equatorial coordinates refer to the standard equinox of 2000.
- The selection criteria for the considered color indices are

$$S.C.BV = |(B-V)_{obs} - (B-V)_{cal}| \leq 0.07 \tag{19}$$

$$S.C.UB = |(U-B)_{obs} - (U-B)_{cal}| \leq 0.12 \tag{20}$$

$$S.C.VI = |(V-I)_{obs} - (V-I)_{cal}| \leq 0.06 \tag{21}$$

$$S.C.VR = |(V-R)_{obs} - (V-R)_{cal}| \leq 0.04 \tag{22}$$

where, for example $(B\text{-}V)_{obs}$ and $(B\text{-}V)_{cal}$ are respectively, the observed (tabulated] and the calculated [from Equation (6)] color indices

- As a result of the criteria (19) to (22) the number of cluster was reduced to 56, 65, 34, 46 for the color indices (*B-V*), (*U-B*), (*V-I*) and (*V-R*) respectively. The corresponding elements and the tolerances of an accurate acceptable solution set are listed in Table 5.

TABLES 5. Elements of an acceptable solution set.

System	C_1	C_2	e_{C_1}	e_{C_2}	e	Q	r	e_{C_1}/C_1	e_{C_2}/C_2
B-V	0.58234	0.06238	0.00518	0.0007	0.02909	0.000602	0.99275	0.00877	0.0113
U-B	0.03391	0.0724	0.00835	0.00136	0.04226	0.00016	0.97655	0.24619	0.01889
V-I	0.79823	0.08155	0.00726	0.0014	0.02583	0.00012	0.98989	0.00911	0.01686
V-R	0.413	0.0332	0.004	0.0006	0.016	0.00004	0.984	0.0094	0.0184

● The selected data corresponding to the selection criteria (3.2) to (3.5) are listed in Tables 6-9.

TABLE 6. Data of the selected globular clusters for *B-V*.

Name	α^o	δ^o	b^o	*B-V*
NGC 288	0.879	– 26.59	– 89.38	0.94
NGC 1261	3.2	– 55.22	– 52.13	0.93
NGC 1904 M79	5.4	– 24.53	– 29.35	0.91
NGC 2298	6.82	– 36	– 16.01	1.11
NGC 2808	9.2	– 64.87	– 11.25	1.18
NGC 4590 M68	12.66	– 26.75	36.05	0.94
NGC 4833	12.97	28.4	– 8.01	1.33
NGC 5024 M53	13.22	18.17	79.76	0.86
MGC 5053	13.7	17.7	78.94	0.9
MGC 5272 M3	13.7	28.37	78.71	0.93
NGC 5286	13.77	– 51.37	10.57	1.19
MGC 5634	14.5	– 5.97	49.26	0.96
NGC 5694	14.66	– 26.53	30.36	0.98
NGC 5824	15.06	– 33.07	22.07	1.05
NGC 5904 M5	15.3	2.07	46.8	0.95
NGC 6144	16.45	– 26.03	15.7	1.1
NGC 6273 M19	17.04	– 26.27	9.38	1.35
NGC 628	17.08	– 24.8	9.94	1.31
NGC 6304	17.23	– 29.46	5.38	1.7
NGC 6388	17.6	– 44.73	6.74	1.47
NGC 6453	17.85	–34.6	– 3.87	2
NGC 6584	18.31	– 52.22	– 16.41	1.04
NGC 6624	18.39	– 30.36	– 7.91	1.42
NGC 6638	18.5	– 25.5	– 7.15	1.5
NGC 6637 M69	18.52	– 32.33	– 10.27	1.28
NGC 6656 M22	18.6	– 23.77	– 7.55	1.42
NGC 6715 M54	18.92	– 30.48	– 14.09	1.1
NGC 6723	18.97	– 36.63	– 17.3	1.05
NGC 6752	18.18	– 59.98	– 25.63	0.93
NGC 6809 M55	19.67	– 30.97	– 23.27	1.28
NGC 6934	20.67	7.4	– 18.89	0.99
NGC 6981 M72	20.98	– 12.52	– 32.68	0.94
NGC 7089 M2	21.55	– 0.82	– 35.78	0.92
NGC 7099 M30	21.67	– 23.18	– 46.83	0.86

TABLE 7. Data of the selected globular clusters for *V-R*.

Name	α^o	δ^o	b^o	*V-R*
NGC 288	0.879	– 26.59	– 89.38	0.45
NGC 362	1.05	– 70.85	– 46.25	0.49
NGC 1261	3.2	– 55.22	– 52.13	0.45
NGC 1851	5.23	– 40.05	– 35.04	0.49
NGC 2298	6.82	– 36	– 16.01	0.45
NGC 2808	9.2	– 64.87	– 11.25	0.57
NGC 3201	10.3	– 46.41	– 8.64	0.62
NGC 4147	12.16	18.5	77.19	0.42
NGC 4590 M68	12.66	– 26.75	36.05	0.46
NGC 4833	12.98	– 70.87	– 8.01	0.63
NGC 5024 M53	13.45	18.16	79.76	0.45
NGC 5139 OMEGA CEN	13.44	– 47.48	14.97	0.51
NGC 5286	13.77	– 51.37	10.57	0.58
NGC 5634	14.5	– 5.97	49.26	0.44
NGC 5694	14.66	– 26.53	30.36	0.48
NGC 5824	15.06	– 33.07	22.07	0.51
NGC 5897	15.29	– 21	30.29	0.5
NGC 5904 M5	15.3	2.07	46.8	0.45
NGC 5927	15.47	– 50.67	4.86	0.79
NGC 5986	15.77	– 37.78	13.27	0.58
NGC 6101	16.43	– 72.22	– 15.82	0.5
NGC 6254 M10	16.95	– 4.1	23.08	0.53
NGC 6273 M19	17.04	– 26.27	9.38	0.66
NGC 6139	16.46	– 38.85	9.94	0.65
NGC 6293	17.17	– 26.58	7.83	0.63
NGC 6304	17.23	– 29.46	5.38	0.77
NGC 6333 M9	17.32	– 18.52	10.7	0.63
NGC 6352	14.42	– 48.42	– 7.17	0.66
NGC 6362	14.42	– 48.42	– 17.57	0.56
NGC 6388	18.5	– 25.5	– 6.74	0.71
NGC 6441	17.83	– 37.05	– 5.01	0.79
NGC 6528	18.08	– 30.06	– 4.17	0.9
NGC 6553	18.15	– 25.9	– 3.02	1.01
NGC 6558	18.17	– 31.76	– 6.03	0.71
NGC 6584	18.31	– 52.22	– 16.41	0.5
NGC 6624	18.39	– 30.36	– 7.91	0.67
NGC 6638	18.5	– 29.5	– 7.15	0.72
NGC 6637 M69	18.52	– 32.33	– 10.27	0.62
NGC 6642	18.53	– 23.48	– 6.44	0.72
NGC 6656 M22	18.6	– 32.98	– 7.55	0.68
NGC 6715 M54	18.92	– 30.48	– 14.09	0.53
NGC 6723	18.97	– 36.63	– 17.3	0.5
NGC 6809 M55	19.67	– 30.97	– 23.27	0.48
NGC 6934	20.67	7.4	– 18.89	0.49
NGC 6981 M72	20.67	7.4	– 32.68	0.45
NGC 7099 M2	21.55	– 0.82	– 35.78	0.46

TABLE 8. Data of the selected globular clusters for *U-B*.

Name	α^{o}	δ^{o}	b^{o}	*U-B*
NGC 288	0.879	– 26.59	– 89.38	0.08
NGC 362	1.05	– 70.85	– 46.25	0.16
NGC 1261	3.2	– 55.22	– 52.13	0.13
NGC 1851	5.23	– 40.05	– 35.04	0.17
NGC 1904 M79	5.4	– 24.53	– 29.35	0.06
NGC 2298	6.82	– 36	– 16.01	0.17
NGC 2419	7.6	38.88	25.24	0.07
NGC 2808	9.2	– 64.87	– 11.25	0.28
NGC 3201	10.3	– 46.41	80.64	0.38
NGC 4147	12.16	18.5	77.19	0.11
NGC 4372	12	– 72.65	– 9.88	0.31
NGC 4590 M68	12.66	– 26.75	36.05	0.04
NGC 5024 M53	13.45	–47.48	79.76	0.09
NGC 5053	13.7	17.7	78.94	0.03
NGC 5139 OMEGA	13.44	– 47.48	14.97	0.19
NGC 5272 M3	13.7	28.37	– 89.38	0.08
NGC 5286	13.77	– 51.37	78.71	0.09
NGC 5466	14.09	28.52	10.57	0.28
NGC 5634	14.5	– 5.97	49.26	0.09
NGC 5694	14.66	– 26.53	30.36	0.08
NGC 5824	15.06	– 33.07	22.07	0.12
NGC 5897	15.29	– 21	30.29	0.08
NGC 5904 M5	15.3	2.07	46.8	0.17
NGC 5927	15.47	– 50.67	4.86	0.85
NGC 5986	15.77	– 37.78	13.27	0.3
NGC 6093 M80	16.28	– 22.97	19.46	0.21
NGC 6144	16.45	– 26.03	15.7	35
NGC 6205 M13	16.69	36.46	40.91	– 0.02
NGC 6229	16.78	47.52	40.31	– 0.03
NGC 6218 M12	16.76	– 1.95	26.31	0.2
NGC 6254 M10	16.95	– 4.1	23.08	0.23
NGC 6266 M62	17.02	– 30.1	7.32	0.52
NGC 6273 M19	17.04	– 26.27	9.38	0.35
NGC 6284	17.08	– 24.75	9.94	0.4
NGC 6304	17.23	– 29.46	5.38	0.82
NGC 6316	17.27	– 28.14	5.76	0.62
NGC 6341 M92	17.28	43.14	34.86	0.01
NGC 6333 M9	17.32	– 18.52	10.7	0.32
NGC 6355	17.4	– 26.35	5.43	0.72
NGC 6352	17.42	– 48.42	7.17	0.64
NGC 6362	17.52	– 67.05	– 17.57	0.29
NGC 6388	17.6	– 44.73	– 6.74	0.66
NGC 6401	17.64	– 23.91	3.98	0.93
NGC 6441	17.83	– 37.05	– 5.01	0.81
NGC 6496	17.98	– 44.25	– 10.01	0.45

TABLE 8. Contd.

Name	α°	δ°	b°	*U-B*
NGC 6535	18.06	– 0.29	10.44	0.29
NGC 6553	18.15	– 25.9	– 3.02	1.34
NGC 6558	18.17	– 31.76	– 6.03	0.58
NGC 6584	18.31	– 52.22	– 16.41	0.17
NGC 6624	18.39	– 30.36	– 7.91	0.6
NGC 6638	18.5	– 25.5	– 7.15	0.56
NGC 6637 M19	18.52	– 32.33	– 10.27	0.48
NGC 6642	18.53	– 23.48	– 6.44	0.57
NGC 6652	18.6	– 32.98	– 11.38	0.37
NGC 6715 M54	18.92	– 30.48	– 14.09	0.24
NGC 6717 Pa19	18.93	– 22.7	– 10.9	0.35
NGC 6723	18.97	– 36.63	– 17.3	0.21
NGC 6752	19.16	– 59.98	– 25.63	0.07
NGC 6760	19.19	1.017	– 3.92	1.09
NGC 6809 M55	19.67	– 30.97	– 23.27	0.11
NGC 6934	20.67	7.4	– 18.89	0.2
NGC 6981 M72	20.91	– 12.52	– 32.68	0.14
NGC 7078 M15	21.5	12.16	– 27.31	0.06
NGC 7089 M2	21.55	– 0.82	– 35.78	0.09
NGC 7099 M30	21.67	– 23.18	– 46.83	0.03

TABLE 9. Data of the selected globular clusters for *V-I*.

Name	α°	δ°	b°	*V-I*
NGC 288	0.879	– 26.59	– 89.38	0.94
NGC 1261	3.2	– 55.22	– 52.13	0.93
NGC 1904 M79	5.4	– 24.53	– 29.35	0.91
NGC 2298	6.82	– 36	– 16.01	1.11
NGC 2808	9.2	– 64.87	– 11.25	1.18
NGC 4590 M68	12.66	17.7	36.05	0.94
NGC 4833	12.97	28.4	– 8.01	1.33
NGC 5024 M53	13.22	18.17	79.76	0.86
MGC 5053	13.7	17.7	78.94	0.9
MGC 5272 M3	13.7	28.37	78.71	0.93
NGC 5286	13.77	– 51.37	10.57	1.19
MGC 5634	14.5	– 5.97	49.26	0.96
NGC 5694	14.66	– 26.53	30.36	0.98
NGC 5824	15.06	– 33.07	22.07	1.05
NGC 5904 M5	15.3	2.07	46.8	0.95
NGC 6144	16.45	– 26.03	15.7	1.1
NGC 6273 M19	17.04	– 26.27	9.38	1.35
NGC 628	17.08	– 24.8	9.94	1.31
NGC 6304	17.23	– 29.46	5.38	1.7
NGC 6388	17.6	– 44.73	6.74	1.47

TABLE 9. Contd.

Name	α^o	δ^o	b^o	V-I
NGC 6453	17.85	– 34.6	– 3.87	2
NGC 6584	18.31	– 52.22	– 16.41	1.04
NGC 6624	18.39	– 30.36	– 7.91	1.42
NGC 6638	18.5	– 25.5	– 7.15	1.5
NGC 6637 M69	18.52	– 32.33	– 10.27	1.28
NGC 6656 M22	18.6	– 23.77	– 7.55	1.42
NGC 6715 M54	18.92	– 30.48	– 14.09	1.1
NGC 6723	18.97	– 36.63	– 17.3	1.05
NGC 6752	18.18	– 59.98	– 25.63	0.93
NGC 6809 M55	19.67	– 30.97	– 23.27	1.28
NGC 6934	20.67	7.4	– 18.89	0.99
NGC 6981 M72	20.98	– 12.52	– 32.68	0.94
NGC 7089 M2	21.55	–0.82	– 35.78	0.92
NGC 7099 M30	21.67	– 23.18	– 46.83	0.86

● The dependence of the solutions and the tolerances ε are listed in Tables 1 to 4 for the different color indices. Also the error bars of Figures 2-5 are given as typical examples of this dependence.

● Figures 5-8 represent the variations of the color indices and ξ = csc | b | for the selected data [of Tables 6-9] together with the corresponding correlation coefficients.

Applications

Determination of the Effective Thickness of the Galactic Absorbing Layer

The coefficient of differential absorption Δb_v and the absorption of coefficient K_v are related by $kv = 3\Delta b_v$[2], where the value of k_v = 1.9 mag/kpc[5]. Hence Δk_{bv} = 1.9/3. Now, according to Equation (7), $C_2 = \Delta b_{v*}h$ since the value of C_2 for B-V is 0.063278 (from Table 5), therefore, the effective thickness $\tau = 2h$ of the galactic absorbing layer is

$$[\tau = 199.825262 = 200\, pc] \qquad (23)$$

which is exactly the same adopted value[6].

Determination of Unknown Value of the Coefficient of Differential Absorption

If we adopt for h the value of 100 pc, then we can use the value of C_2 to compute from Equation (7) the corresponding Δts from

$$\Delta ts \;=\; C_{2*}\, 10 \text{ mag/kpc}$$

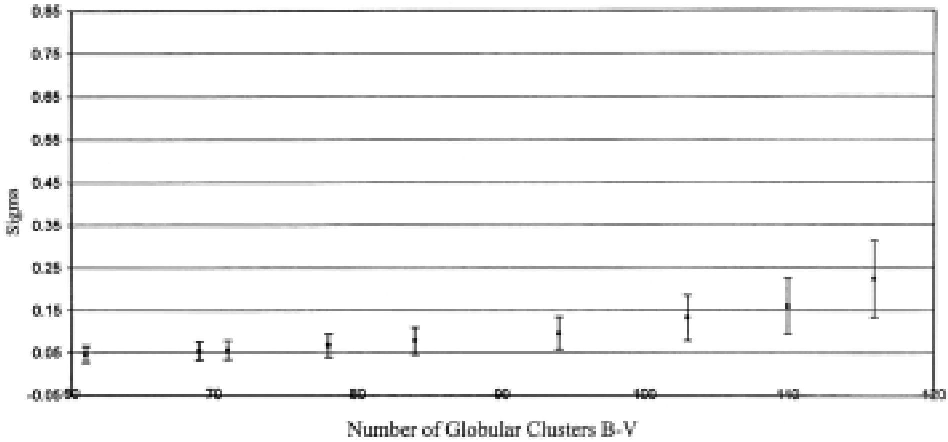

FIG. 2. Error bars are indicated for σ.

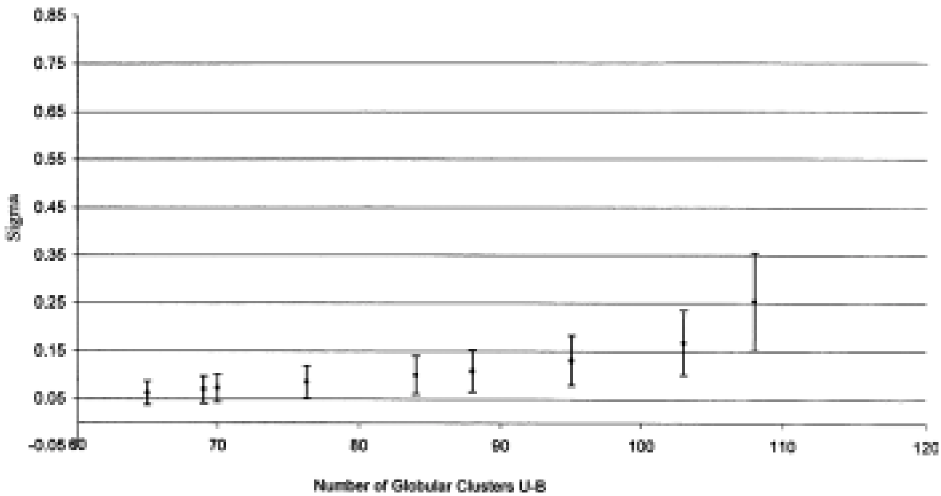

FIG. 3. Error bars are indicated for σ.

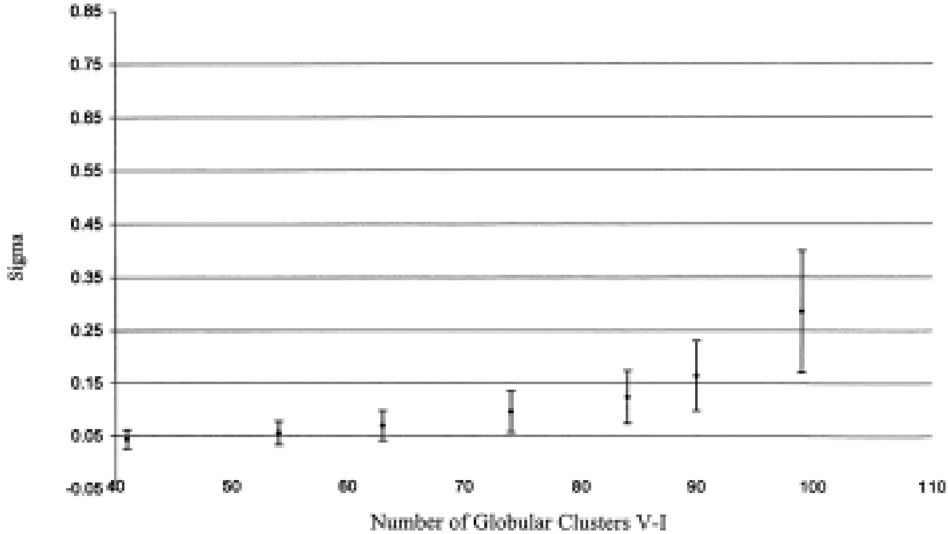

FIG. 4. Error bars are indicated for σ.

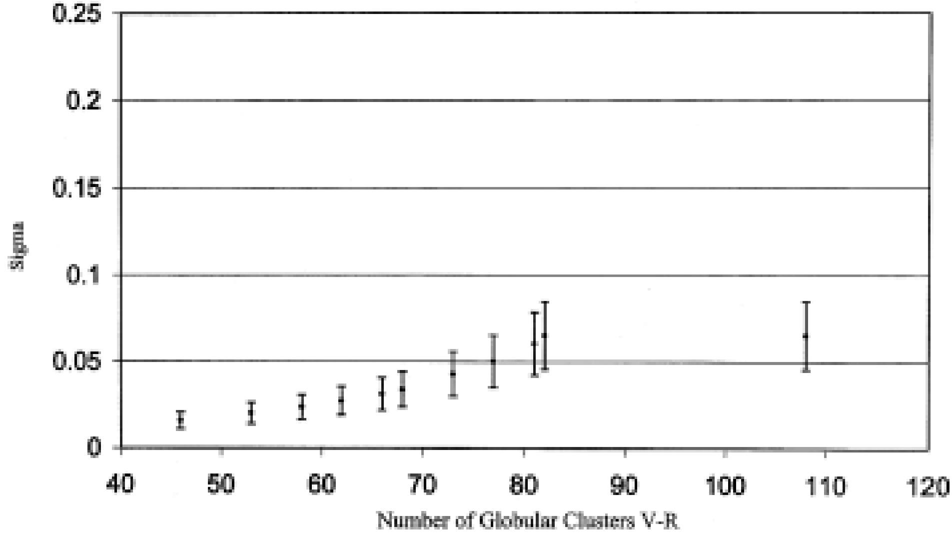

FIG. 5. Error bars are indicated for σ.

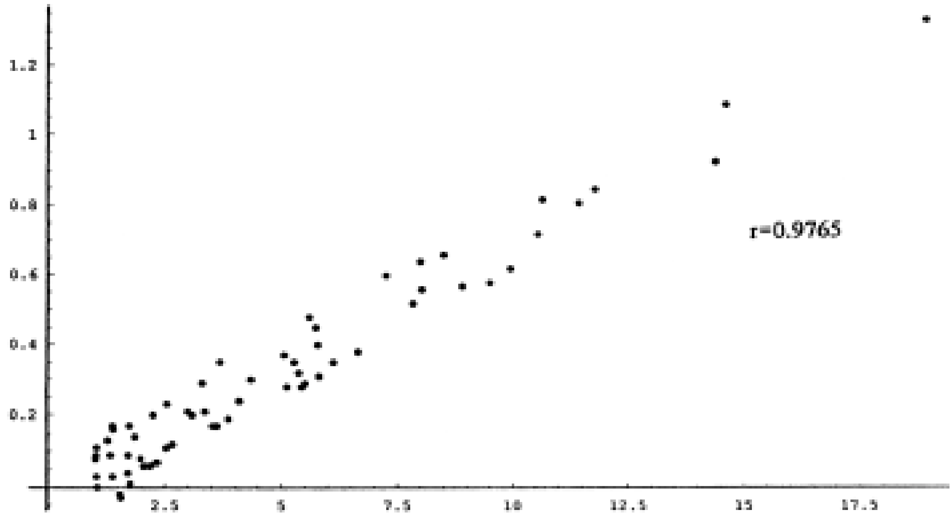

FIG. 6. The variation of (*U-B*) with csc | *b* |.

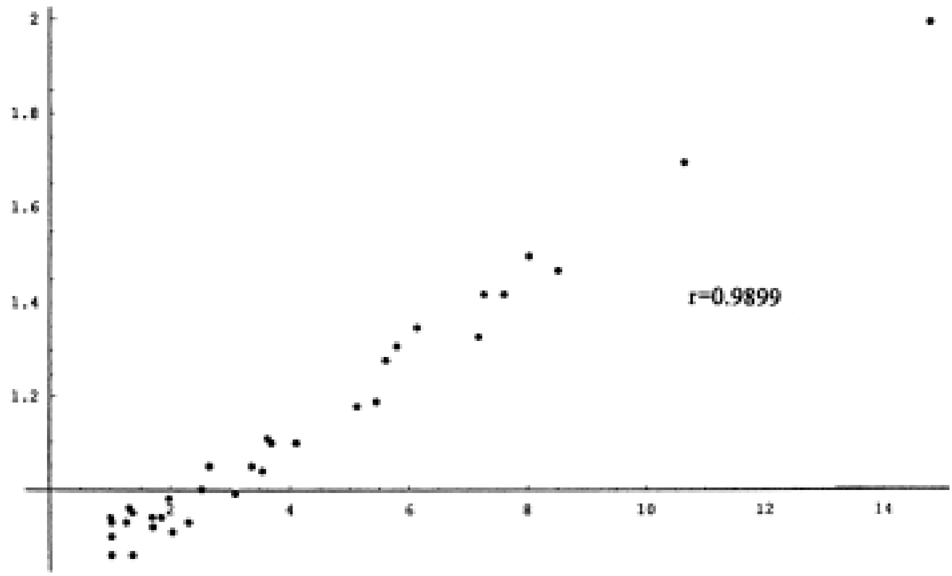

FIG. 7. The variation of (*V-I*) with csc | *b* |.

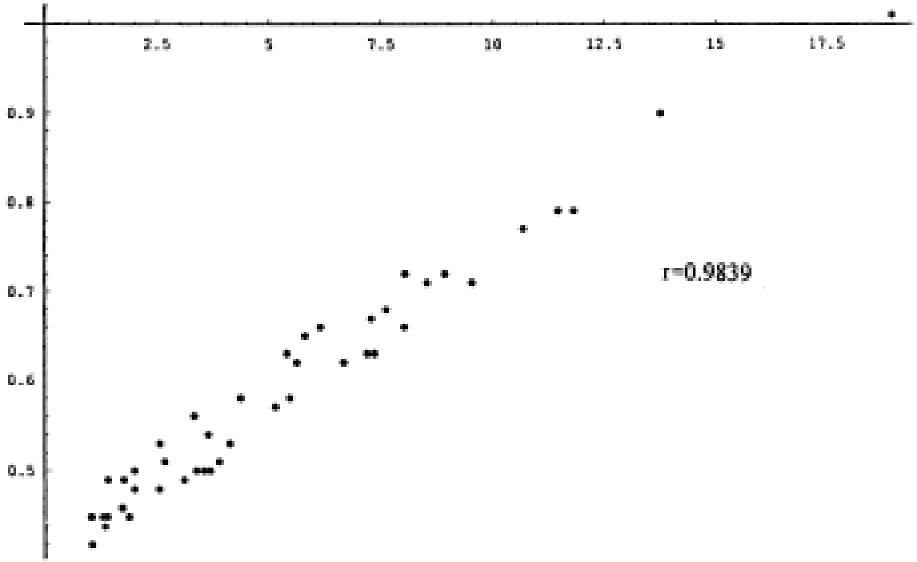

FIG. 8. The variation of $(V\text{-}R)$ with csc $|b|$.

As applications of this formula, we consider the value of C_2 of Table 5 to compute,

$$\Delta_{ub} = k_u - k_b = 0.724 \text{ mag / kpc}, \tag{24}$$

$$\Delta_{vi} = k_v - k_i = 0.81549 \text{ mag / kpc}, \tag{25}$$

$$\Delta_{vr} = k_v - k_r = 0.33189 \text{ mag / kpc}, \tag{26}$$

In concluding the present paper, selection and solution criteria enable us to determine very accurately the intrinsic color indices of globular clusters from the linear representation of the average color index and csc b. These values are $(B\text{-}V)_o$ = 0.582344 mag, $(U\text{-}B)_o$ = 0.033909 mag, $(V\text{-}I)_o$ = 0.798231 mag, $(V\text{-}R)_o$ = 0.413043 mag. From the slope of such linear representation we compute: (1) the effective thickness of the galactic absorbing layer as 200 pc which is exactly the adopted value, and (2) the values of coefficients of differential absorption for *U-B, V-I* and *V-R* systems: $k_u - k_b$ = 0.724 mag/kpc, $k_v - k_i$ = 0.81549 mag/kpc and $k_v - k_r$ = 0.33189 mag/kpc.

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تعيين المعاملات اللونية الذاتية للحشود الكروية ومعاملا ت الامتصاص الجزئية

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المستخلص . لقد تم في هذا البحث استنتاج علاقة خطية للمعامل اللونى المرصود للحشود الكروية في مجرتنا وذلك باستخدام نموذج مبسط لطبقة الامتصاص المجرية 0 كما تمكنا باستخدام معيار الاختيار والحل من تعيين معاملات اللون للحشود الكروية بدقة عالية جداً وهي :

$(B\text{-}V)o$ =0.582344 mag.;$(U\text{-}B)o$ =0.033909 mag;
$(V\text{-}I)o$ =0.798231 mag; $(V\text{-}R)o$ =0.413043 mag

بالإضافة إلى أنه من ميل هذه العلاقة الخطية قمنا بحساب (١) السمُك الحقيقي لطبقة الامتصاص المجري والتي تصل قيمتها 200 PC وهي نفس القيمة المختارة عالميًا ، (٢) قيم معاملات الامتصاص الجزئي وهي :

$k_u\text{-}k_b$ =0.724 mag/kpc, $k_v\text{-}k_i$ = 0.81549 mag/kpc and
$k_v\text{-}k_r$ = 0.33189 mag/kpc.